Seismic analysis of eccentric buildings – An evaluation of existing techniques

René Tinawi & Guy Vachon
Ecole Polytechnique, Montreal, Quebec, Canada

ABSTRACT: The three dimensional seismic response of buildings is presented. A first investigation involving single storey buildings is carried out. The importance of proper idealisation of the mass and specially the mass moment of inertia for three dimensional response is demonstrated when lateral-torsional coupling is important. A second investigation of a four-storey building is used to show the differences in internal load distribution for a fixed base shear using different techniques. Two of these techniques involve different combinations of modal response spectra such as RSS (Root Sum Squared) and CQC (Complete Quadratic Combination). Other static techniques involve the usage of the 1985 NBCC or the application of seismic loads directly at the centre of mass of an eccentric building.

1.0 INTRODUCTION

Seismic analysis of eccentric buildings is a complex problem (Kan 1976, 1977a, 1977b). Recommendations of the 1985 National Building Code of Canada are simple to apply (Tso 1983, Humar 1983, 1984) when the eccentricity is constant throughout the height of the building assuming of course that the centre of rigidity presents no major difficulty in its evaluation. On the other hand when this is a problem in itself (Riddel 1984) a three dimensional dynamic analysis should be performed in order to obtain a more realistic response of the building.

The object of this paper is twofold: The first one is to study the behaviour of three single storey buildings that have respectively double symmetry, single symmetry and no symmetry at all. For these simple buildings the importance of mass idealisation for the torsional response will be highlighted due to the fact that commercially available computer programs lump masses at all the joints. This procedure has the advantage of saving the designer the burden of evaluating the structural mass. On the other hand it does tend to increase the number of dynamic degrees of freedom. Furthermore, floor slabs are often considered rigid and many commercially available programs do not allow this feature to be introduced easily. NASTRAN for example does allow the introduction of Multi-Point Constraints (MPC) but the constraint equations for each floor have to be derived by the designer and introduced into the mathematical model.

Alternatively if the designer does not want to be bothered with such lengthy procedures, a system of "rigid" massless horizontal bracing could be forced to simulate a rigid diaphragm. Either method for simulation of a rigid horizontal diaphragm would enable the designer to allocate only three dynamic degrees of freedom (two translations and one rotation) at the center of mass of each floor. Needless to say that the translational mass calculated either through automated lumping at the joints or by using rigid diaphragms will always be the same. However, the exact mass moment of inertia I_o at the center of mass calculated on the basis of a rigid slab is not necessarily equal to $\Sigma m_i r_i^2$ where m_i represents the lumped mass and r_i the distance from the center of mass.

The first objective of the paper is precisely to evaluate the effect of

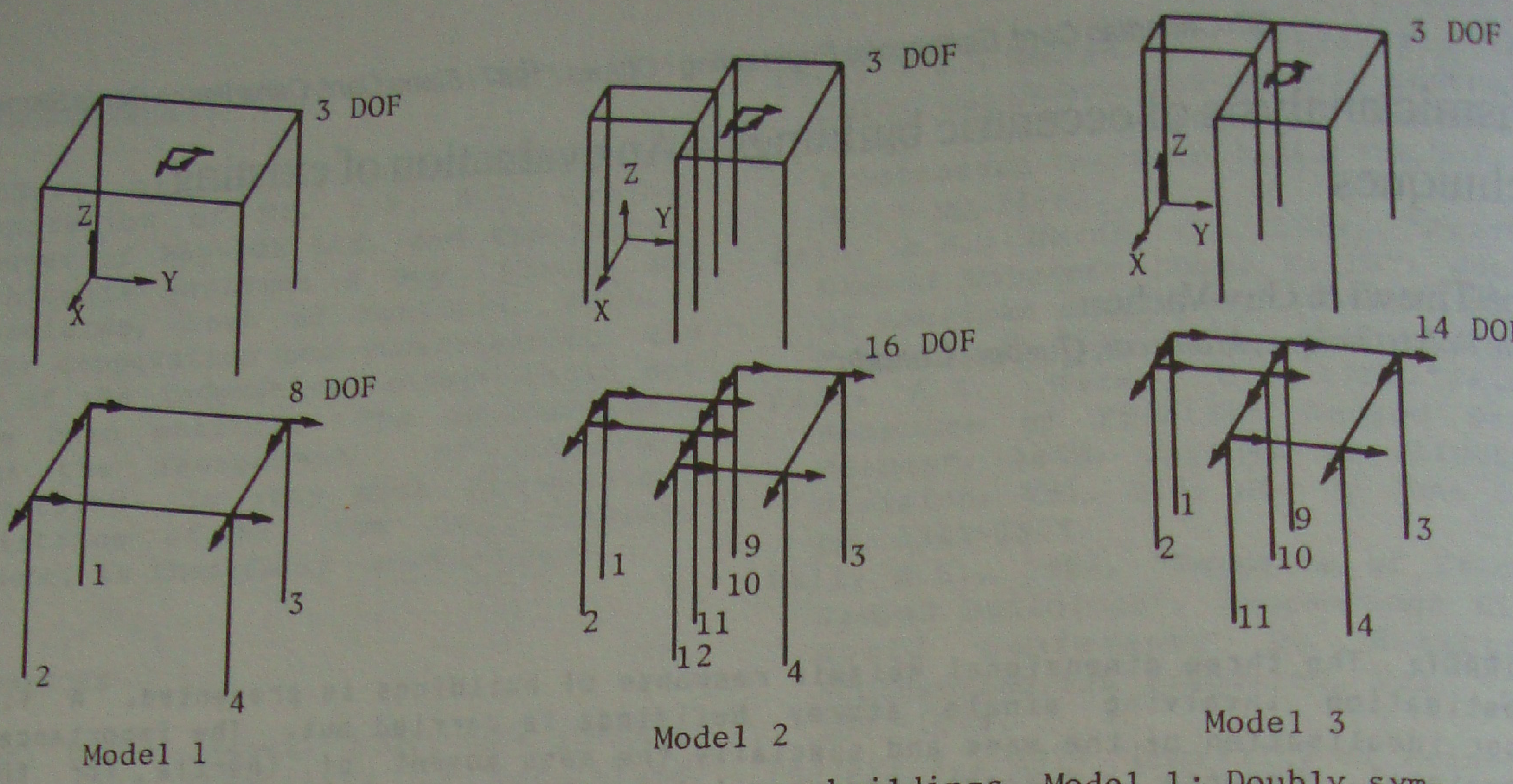

Figure 1. Mass idealisation for single storey buildings. Model 1: Doubly symmetric. Model 2: Single symmetry. Model 3: No symmetry.

idealising the torsional mass moment of inertia on the overall response of the structure and its effect on the internal distribution of the seismic forces. In order to eliminate the zoning effect, acceleration and velocity levels, the normalized distribution spectrum shown in Commentary J to the NBCC85 was used.

The second part of the paper is concerned about comparing internal load distribution in two eccentric four storey buildings using two dynamic techniques and three static methods.

The two models were identical in geometry. However, the first one had a fundamental flexural period of .28s while the second had a period of .87s. Results for the two buildings using different eccentricity to width ratios will be presented for both models thus emphasizing the effect of the constant acceleration and constant velocity spectrum curves.

2.0 SINGLE STOREY BUILDINGS

2.1 Description of the models

Figure 1 shows three single storey buildings that have respectively double, single and no symmetry. For each model, two idealisations for the masses have been performed. In the first case three dynamic degrees of freedom have been allocated at the center of mass. In the second case, lumping masses at top of columns yielded respectively for models 1, 2 and 3 eight, sixteen and forteen translational degrees of freedom.

In order to introduce a deliberate eccentricity, a percentage of the stiffness in the X-direction of columns 1-2 has been transfered to 3-4 while keeping the total stiffness and mass idealisation identical. As for the stiffness in the Y-direction, it was not modified. The earthquake direction was assumed to be in the X-direction only.

2.2 Mode shapes

For each model, a transfer of rigidity from frame 1-2 to 3-4 was such that purely decoupled modes of vibration were obtained. As seen from figure 2, only the first three modes are shown for each model: pure translation in X and Y and pure rotation about Z axis. In order to obtain an uncoupled behaviour for model 2, 50% of the stiffness was transfered to frame 3-4 while for model 3 the corresponding value is 43%. For values other than the ones mentioned, flexural-torsional coupling is clearly shown. However, flexural modes in the Y-direction are not at all affected.

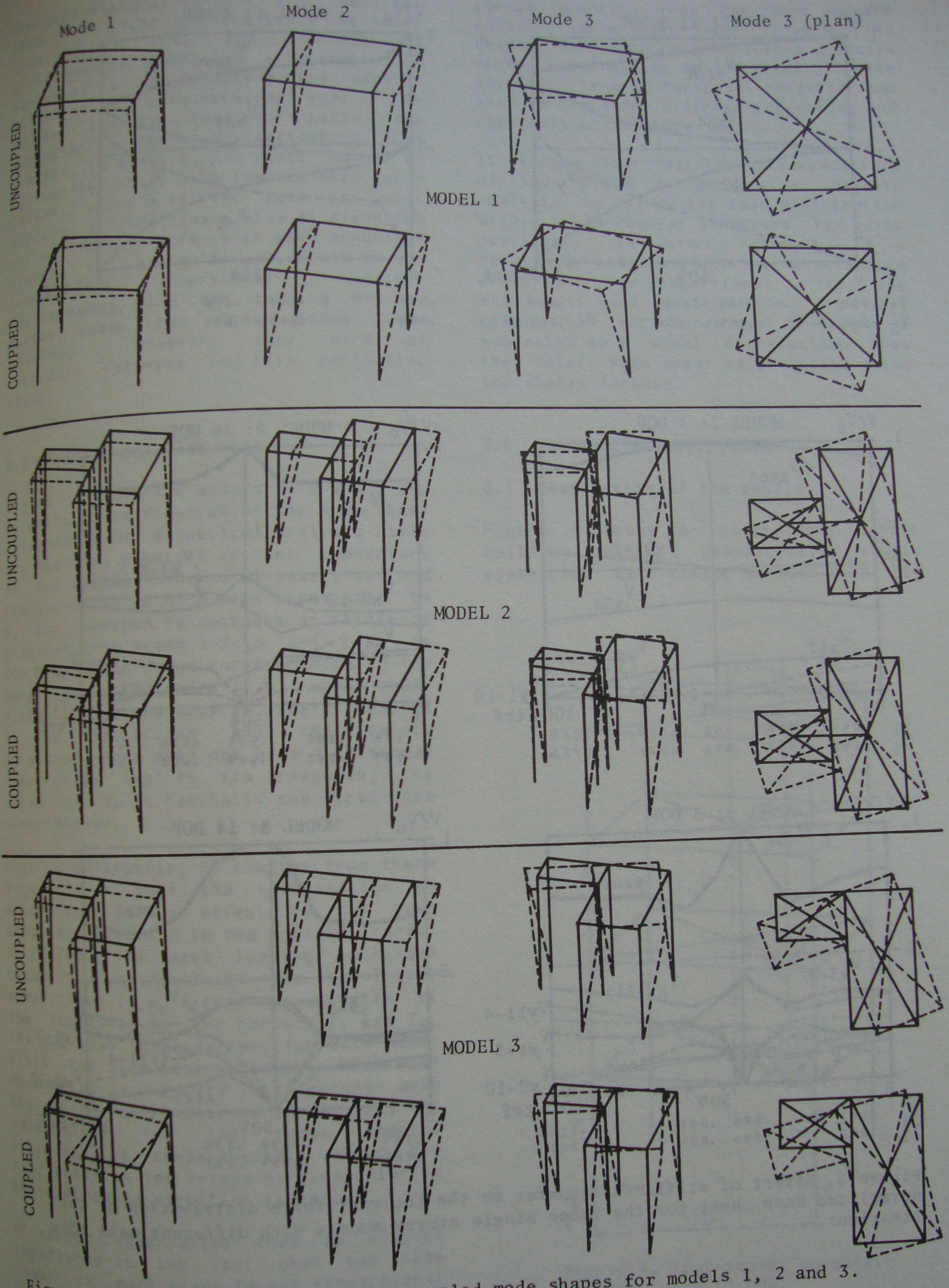

Figure 2. First three coupled and uncoupled mode shapes for models 1, 2 and 3.

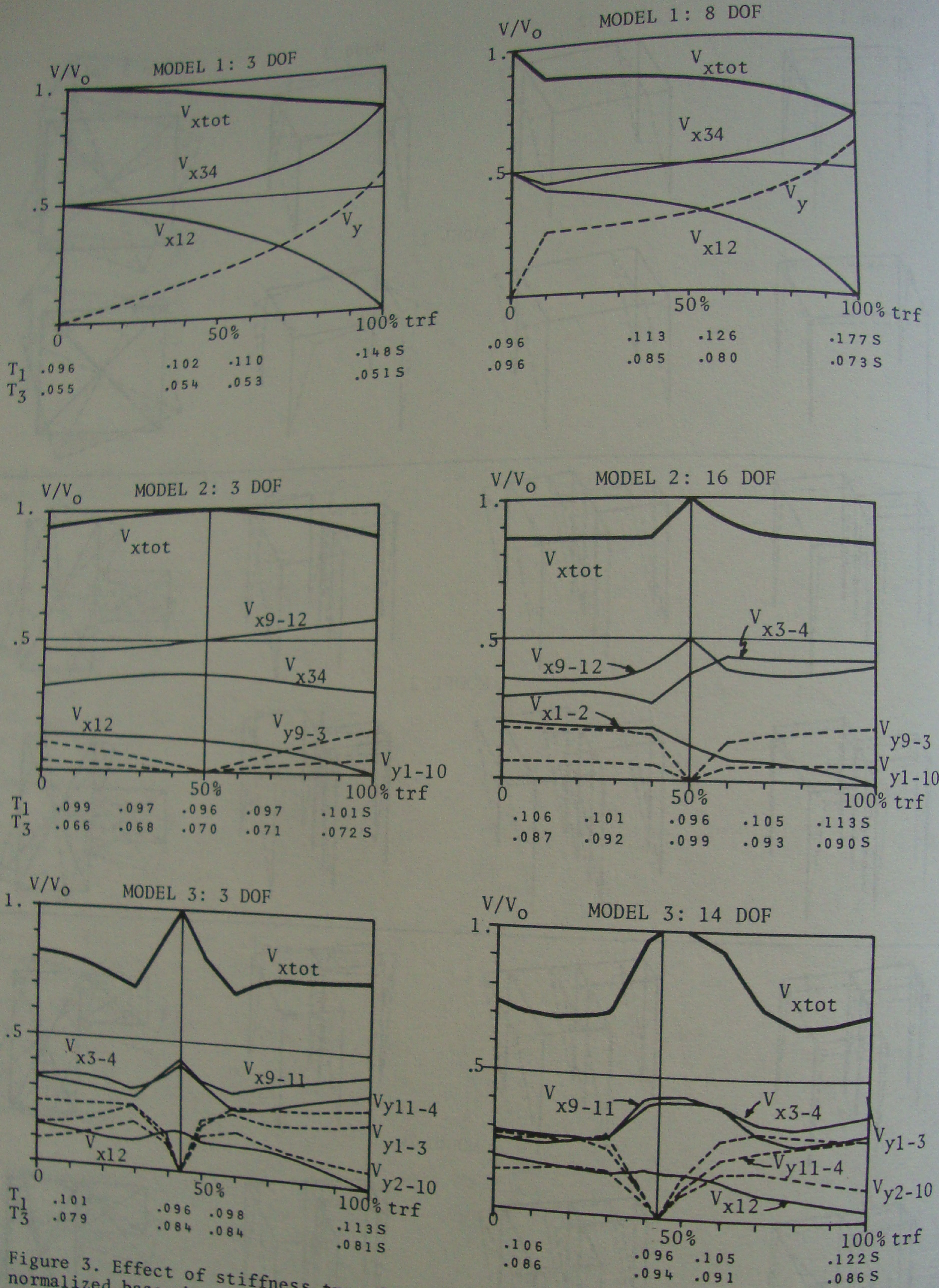

Figure 3. Effect of stiffness transfer on the internal force distribution of the normalized base shear for the three single storey models with different mass idealisations.

The pure flexural periods are identical when there is no coupling since the total translational mass and corresponding stiffness is identical irrespective of the means of distributing the masses. However, the pure torsional mode or the coupled flexural torsional modes are dependant on the mass idealisation. For example, in model 1, the exact value of $I_o = ma^4/6$ while with lumping this value is $ma^4/2$ where m is the mass per unit surface of the square slab of dimension a x a. This difference in mass moment of inertia has a significant effect on the fundamental torsional period. Furthermore, with the lumping of the masses, more than three modes were obtained. However, they were of secondary interest in this particular study.

2.3 Results and discussion

Figure 3 shows for models 1, 2 and 3 the normalized distribution of the base shear obtained from a spectral analysis using the spectrum provided in the Commentary of the NBCC85. The total base shear and its distribution have been normalized to V_o for various percentages of stiffness transfer from frame 1-2 to 3-4. V_o is the base shear value corresponding to the purely decoupled modes (i.e. when e=0). Since the second mode is purely flexural in the Y-direction and the purely torsional mode does not have a torsional spectrum to amplify its response, the value of V_o is basically the first mode contribution.

What is interesting to observe from these figures is that the idealisation of masses does have an effect on the base shear distribution in the various frames. In reality, the mass lumping technique tends to underestimate the total base shear. This is a direct consequence of the increase in the periods T_1 and T_3. The two mass idealisations for each model yield the same base shear only when mode decoupling is evident. This occurs when the stiffness transfer is 0%, 50% and 43% respectively for the three models. Obviously at these specific values, the base shear in the frames perpendicular to the earthquake direction is zero.

A second observation that is also of importance is the fact that the base shear in each frame is not proportional to its stiffness. Otherwise, all the curves starting from the point of zero coupling would have been linear. The reason for such departure from a simple linear distribution is due first to the coupled flexural-torsional behaviour and also to the modal distribution method and combination technique (RSS).

It becomes clear that the recommendations of the NBCC85 to perform a dynamic analysis to study the load distribution within a non-regular structure is quite justified. In such a context, if a relatively simple single storey structure exhibits non-proportional (to the stiffness) load distribution, the very complex 3D structures must therefore be subjected to a modal distribution when the total base shear is evaluated using the static formula.

3.0 MULTI-STOREY BUILDINGS

3.1 Description of the models

Figure 4 shows a four-storey shear building that is geometrically doubly symmetric. Each center of mass with its

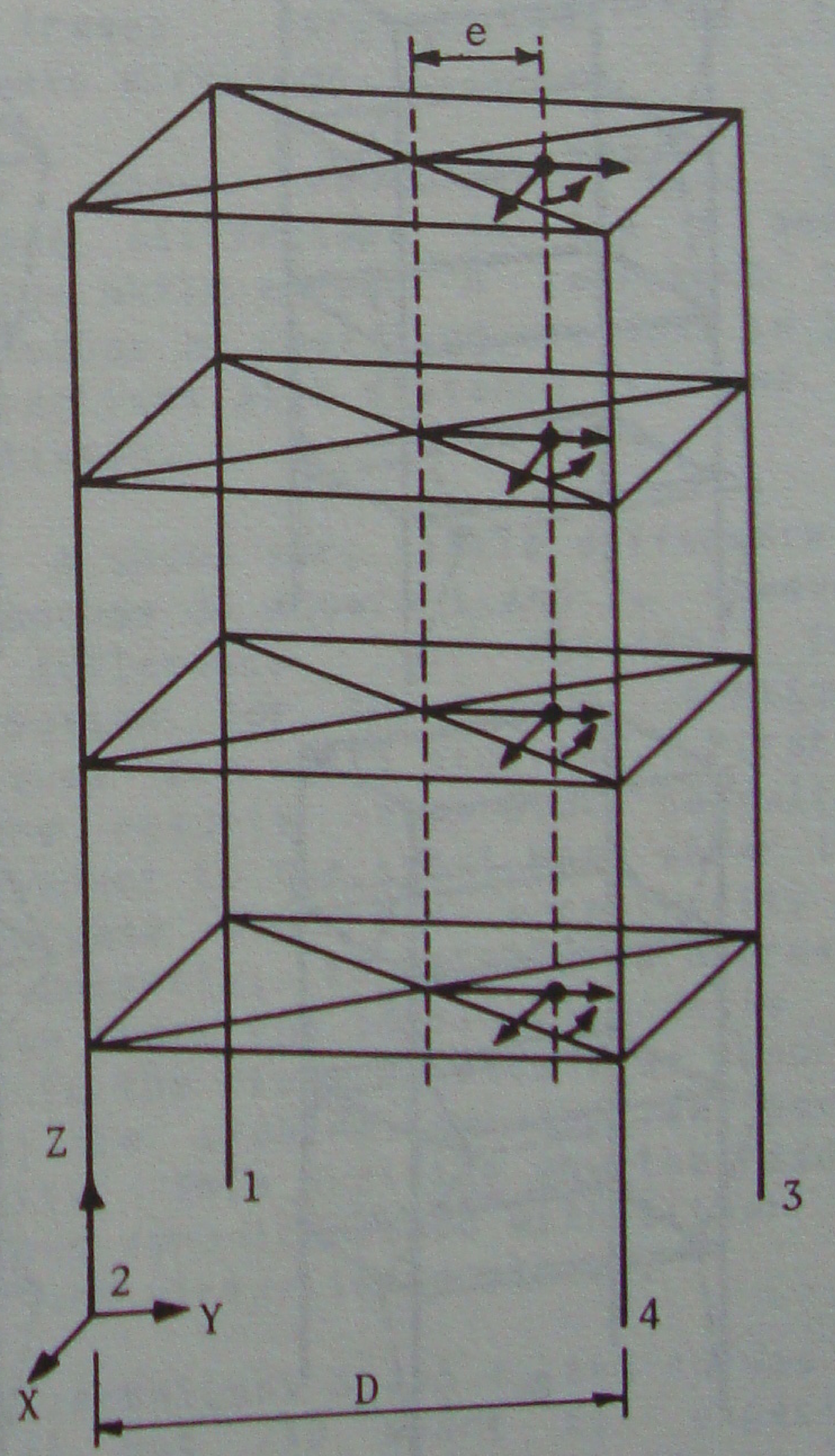

Figure 4. Four-storey eccentric building with 12 DOF.

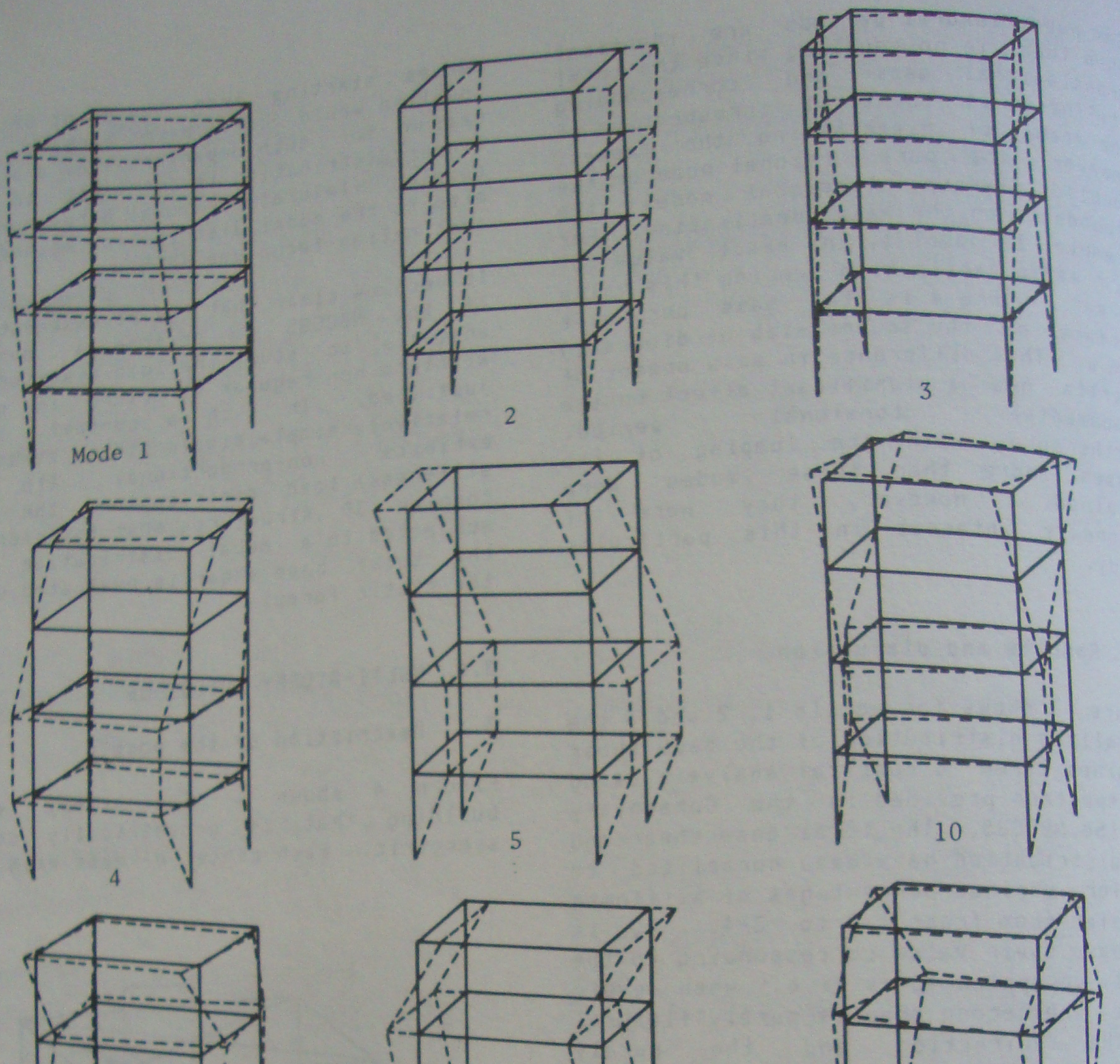

Figure 5. Mode shapes of four-storey building.

three dynamic degrees of freedom has been laterally displaced in the Y-direction thus creating an eccentricity ratio e/D varying from 0 to 25%. The value of the translational mass in the X-direction was varied in order to obtain two models: The first one having a fundamental period of .28s (model 1) while the second one having a fundamental period of .87s (model 2) as mentioned earlier.

In order to limit the number of parameters involved, the value of the mass and that of the mass moment of inertia I_o have been kept constant with respect to the geometric center. Therefore the term Me^2 was substracted from the value of I_o.

3.2 Mode shapes

Shown in figure 5 are the twelve modes of vibration for a typical coupled flexural-torsional behaviour. Modes 2, 5, 7 and 9 are uncoupled flexural modes in the Y-direction and have no contribution to the response for an earthquake acting in the X-direction. Modes 1, 4, 6 and 8 represent the flexural-torsional coupling behaviour while modes 3, 10, 11 and 12 are mainly torsional but coupled with a flexural component.

3.3 Methods of analysis and results

Contrary to the single storey models where only the RSS technique was utilised for combining the modal response, models 1 and 2 of figure 4, were subjected to additional techniques for evaluation of the internal load distribution. In summary these techniques are:

1) Three dimensional spectral analysis with RSS combination for modal response.

2) Same as above except that a Complete Quadratic Combination (Rosenblueth 1968, Wilson 1981) was performed.

3) Static recommendations of the NBCC85 using for the eccentricity $e_x = 1.5e + 0.1D$ where e is the distance between the center of mass and the hypothetical center of rigidity.

4) Similar to 3) except that $e_x = 1.5e$ thus evaluating the validity of the 1.5 amplification factor.

5) Application of the vertical static load distribution of the NBCC85 using:

$$F_x = (V-F_t)\,\frac{W_x h_x}{\Sigma\, W_i h_i}$$

to the center of mass of a 3D static model.

For methods 3) and 4) the internal distribution of the base shear is carried out (Picard 1979) using:

$$V_i = V\left[\frac{k_{xi}}{\sum\limits_{m} k_{xi}} \pm \frac{k_{xi}\, y_i\, e}{\sum\limits_{m} k_{xi}\, y_i^2 + \sum\limits_{n} k_{yi}\, x_i^2}\right]$$

for frames parallel to the earthquake direction; and

$$V_i = V\left[\frac{k_{yi}\, x_i\, e}{\sum\limits_{m} k_{xi}\, y_i^2 + \sum\limits_{n} k_{yi}\, x_i^2}\right]$$

for frames perpendicular to the earthquake direction.

In the above equations, k_{xi}, k_{yi} represent stiffnesses in the X and Y direction while m and n represent the total number of resisting elements in the two directions at a distance x_i and y_i respectively.

Figure 6 shows very little difference in the response of models 1 and 2. However, the difference in internal load distribution for the five mentioned techniques are worth studying. First of all, the results have been normalised with respect to the total base shear (V_o) for the case when the eccentricity is zero. Secondly, these results represent the case where the torsional mode is not close to the flexural mode thus ignoring totally the problem related to period proximity. This explains why the results using the dynamic method with either RSS or CQC are virtually identical.

As for techniques 3 and 4, the curves are parallel but .1D apart as expected. Comparing technique 4 with 1 or 2 it appears that the former does not

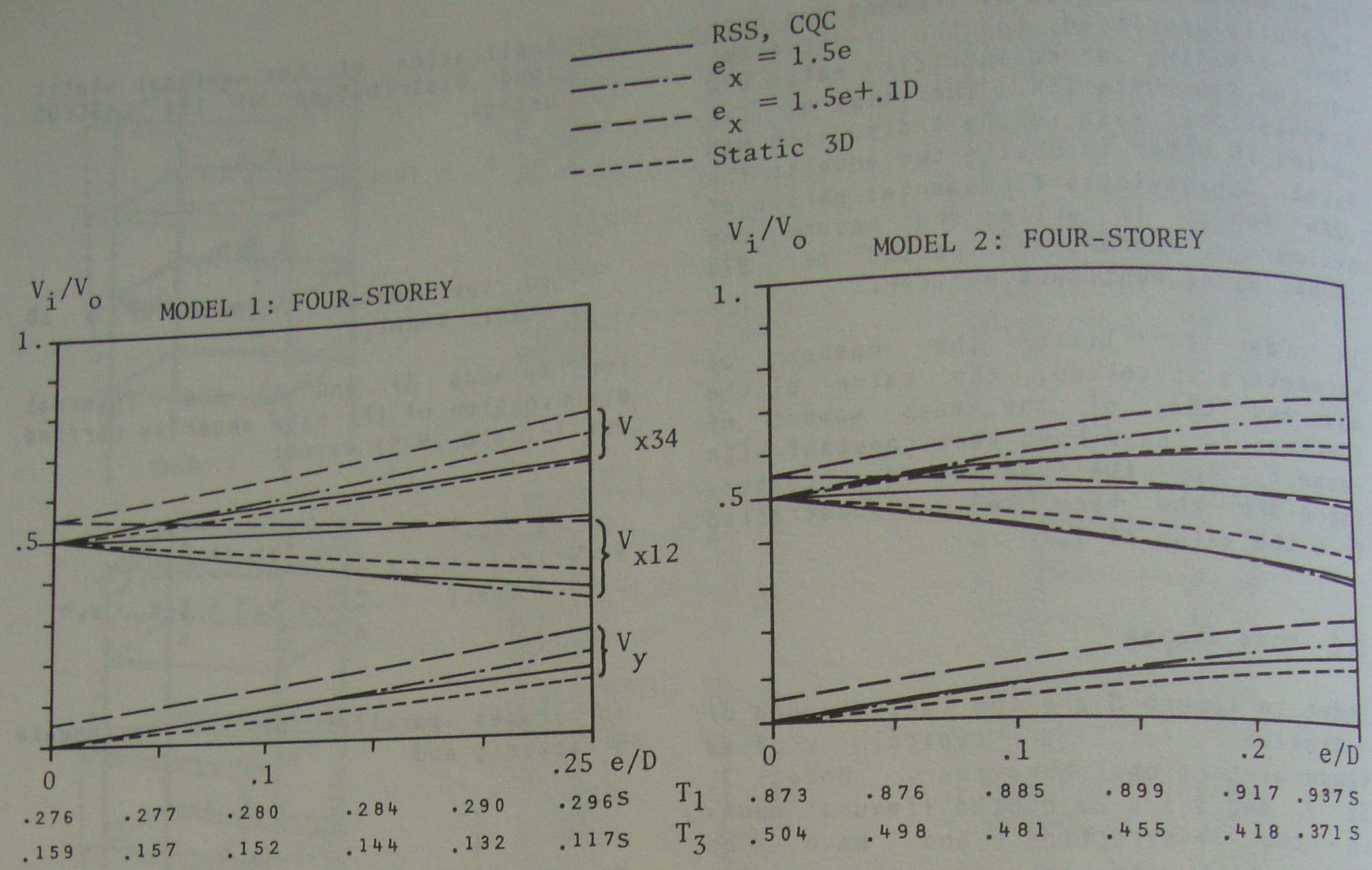

Figure 6. Comparison between various techniques for evaluation of internal force distribution for different e/D ratios. (T_1 is the flexural period and T_3 the torsional one).

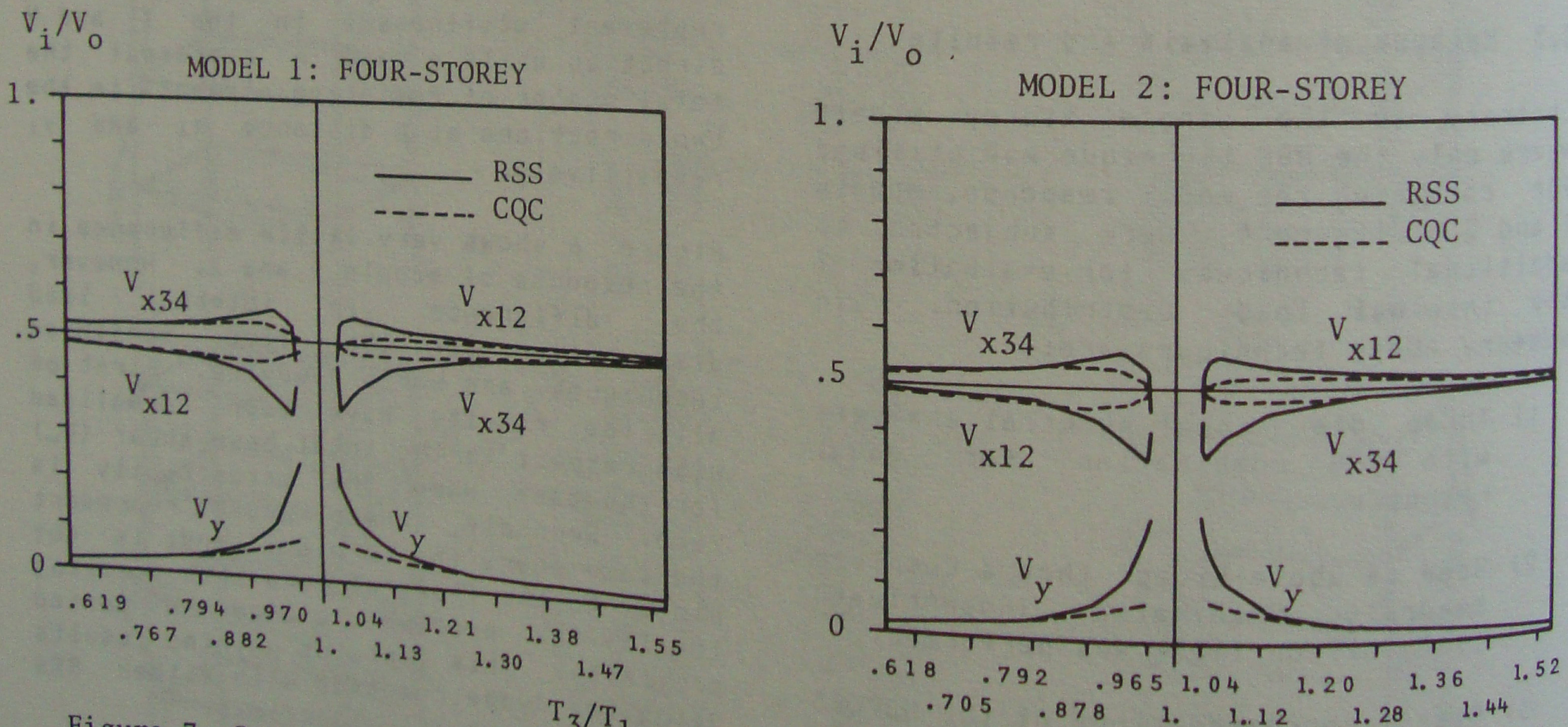

Figure 7. Comparison between RSS and CQC combination techniques for the distribution of internal forces and for different ratios of T_3/T_1.

necessarily overestimate the shear distribution. Technique 5 compares fairly well with the dynamic analysis but is not necessarily conversative. As for values in the frames perpendicular to the earthquake direction, this technique underestimates the response as compared to methods 1 and 2.

In order to investigate the various modal combination techniques proposed when proximity of the flexural and torsional modes is evident, the mass moment of inertia I_{θ} was varied in order to increase arbitrarily the torsional period compared to the fundamental flexural period.

Figure 7 shows clearly the very large differences between the two techniques when the ratio of the periods is between 0.8 and 1.2. Furthermore, it is not obvious which technique will be conservative within this range. One obvious way to eliminate the problem is simply to avoid close proximity of the fundamental flexural and torsional periods since both techniques yield the same results otherwise.

4.0 CONCLUSIONS

Three dimensional dynamic analyses have been performed on three single-storey buildings and two four-storey buildings. Deliberate eccentricities were created by varying the relative stiffness of the lateral resisting elements or by displacing the center of mass away from the geometric center. The following conclusions can be drawn:

1. Automated mass lumping can lead to erroneous values for the mass moment of inertia which in turn can underestimate the total base shear and modify its internal distribution.

2. For a given base shear, the internal distribution of forces is not proportional to the stiffness of the resisting elements.

3. Dynamic analyses performed on two four-storey buildings yield identical results when the modes are combined either through RSS or CQC techniques providing the proximity of the fundamental modes in flexure and torsion are at least 20% apart. Otherwise both techniques could be equally unreliable.

4. For the simple models presented (without accidental eccentricity), the static method for eccentric buildings using the NBCC85 always overestimates the results of internal distribution compared to dynamic methods. On the other hand the usage of the 1.5 amplification factor is somewhat slightly unconservative in certain cases.

5. The application of forces calculated using the NBCC85 and applied to a 3D static model at the center of mass and majored by 1.5 provides a conservative response compared with dynamic methods.

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